Handwritten notes and problems

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Topic - PnC

Permutations and combinations

Exponent of a prime p in $\underline{|n}$.

$$E_p(\underline{|n}) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \dots$$

1. Exponent of 7 in $\underline{|100}$ is
(a) 14 (b) 15 [✓ (c) 16] (d) none of these

2. The number of zeros at the end of $\underline{|100}$ is UD
(a) 23 [✓ (b) 24] (c) 25 (d) none of these

3. The exponent of 12 in $\underline{|100}$ is
[✓ (a) 48] (b) 49 (c) 96 (d) none of these

4. The number $\underline{|24}$ is divisible by
(a) 6^{24} [✓ (b) 24^6] (c) 12^{12} (d) 48^5

5. Exponent of 15 in $\underline{|100}$ is
[✓ (a) 24] (b) 48 (c) 72 (d) none of these

Number of divisors and sum of divisors

If $n = p^a q^b r^c$, where p, q, r are primes and $a, b, c \in \underline{N}$
then no. of divisors $= (a+1)(b+1)(c+1)$ and
sum of divisors $= (1+p+p^2+\dots+p^a)(1+q+q^2+\dots+q^b)(1+r+r^2+\dots+r^c)$

6. The number of proper factors of 75600 is
(a) 120 (b) 119 [✓ (c) 118] (d) none of these

7. If x, y, z, ... are (n+1) prime numbers, then the number of factors of $x^n y z \dots$ are
(a) $n(n+1)$ [✓ (b) $(n+1)2^n$] (c) $2^n(n+1)$ (d) $n\,2^n$

8. The sum of divisors of $2^5 . 3^4 . 5^2$ is
(a) $3^2.7.11^2$ (b) $3.7.11.31$ [✓ (c) $3^2.7.11^2.31$] (d) none of these

9. The number of divisors of $(6!)^{3!}$ is UD
(a) 364 (b) 9100 [✓ (c) 2275] (d) 75

Number of Dearrangements

If n objects are arranged in a row, then the number of ways in which they can be arranged so that none of them occupies the place assigned to it $= \underline{|n}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \dots + (-1)^n \frac{1}{\underline{|n}}\right)$.

P.T.O

10. There are 5 letters and 5 directed envelopes. The number of ways in which all the letters can be put in wrong envelopes is
(a) 119 [✓ (b) 44] (c) 59 (d) 40

11. Mohan write a letter to five of his friends and addresses them. The number of ways in which letters can be placed in envelopes so that three of them are in the wrong envelopes is
(a) 44 (b) 119 (c) 21 [✓ (d) 20] UD

Number of lines, triangles ; ~~and~~ diagonals (in a polygon)

(1) If there are n distinct points in a plane, no three of which are collinear, except m, all of which lie on a line, then
(i) the number of lines which can be drawn by joining them is $^nC_2 - ^mC_2 + 1$
(ii) the number of triangles that can be formed by joining them $= ^nC_3 - ^mC_3$

(2) If a polygon has n sides $(n > 3)$, then the number of diagonals $= ^nC_2 - n = \frac{n(n-3)}{2}$.

12. There are 18 points in a plane such that no three of them are in the same line except 5 points which are collinear.
(i) The number of lines that can be drawn by joining them is
(a) 143 [✓ (b) 144] (c) 145 (d) none of these
(ii) The number of triangles that can be drawn by joining them is
(a) 805 [✓ (b) 806] (c) 816 (d) none of these

13. A polygon has 170 diagonals, the number of its sides is
(a) 12 (b) 17 (c) 20 (d) 25

14. The number of triangles whose vertices are the vertices of an octagon but none of whose sides happen to come from the sides of octagon is
(a) 24 (b) 52 (c) 48 [✓ (d) 16] UD

15. Ten different letters are printed round a table. The number of ways in which we can select three letters so that no two of them are consecutive is
(a) 26 [✓ (b) 50] (c) 56 (d) 72 UD

16. The total number of squares in a chessboard is
(a) 64 (b) 65 [✓ (c) 204] (d) none of these UD

1. How many seven digit numbers can be formed by using only three digits 3, 7, 9; the digit 3 occurring exactly twice in each number? 672

2. How many numbers are there between 100 and 1000 such that atleast one of their digit is 3? 252

3. How many even numbers lying between 200 and 500 can be formed using the digits 1, 2, 3, 4, 5, 6 if no digit appears more than once in any number? 28
(Repitition allowed ? 42)

4. How many 5-digit even numbers can be formed from the digits 0, 1, 2, 3, 4; repetition of digits not allowed ? 60

5. How many 5-digit numbers divisible by 3 can be formed using digits 0, 2, 4, 6, 8, 9, if repetition not allowed ? 192 UD

6. How many 5-digit numbers divisible by 4 can be formed using digits 0, 2, 4, 7, 8, 9, if repetition not allowed ? 228 UD

7. How many 5-digit numbers divisible by 6 can be formed using digits 0, 2, 4, 5, 6, 8, if repetition not allowed ? 78 UD

8. The total number of 4 digit numbers greater than 4000, whose sum of digits is odd is
(a) 2800 [✓ (b) 3000] (c) 3600 (d) none of these UD

9. ~~How many~~ The number of six-digit numbers ~~are there~~ in which the sum of digits is divisible by 5 is
[✓ (a) 180000] (b) 540000 (c) 5×10^5 (d) none of these UD

10. How many different 3 digit numbers can be formed with three four's, four two's and two three's? 26

11. The total number of odd natural numbers that can be formed with the digits 1, 3, 1, 5, 4, 1, 4 which are greater than 2 million are
(a) 120 (b) 160 [✓ (c) 180] (d) none of these UD

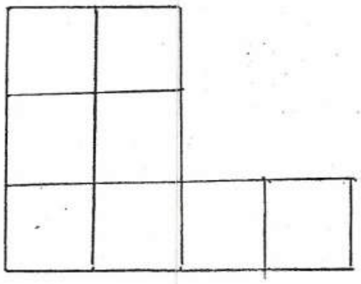
12. Ten IIT and 2 DCE students sit in a row. The number of ways in which 3 IIT students sit in between 2 DCE students is
[✓ (a) ${}^{10}C_3 \times 2! \times 3! \times 8!$] (b) $10! \times 2! \times 3! \times 8!$
(c) $5! \times 2! \times 3! \times 9!$ (d) none of these UD

P.T.O.

13. The number of ways in which we can select four numbers from 1 to 30 so as to exclude every selection of four consecutive numbers is UD

[✓ (a) 27378] (b) 27405 (c) 27399 (d) none of these

14. In how many ways the squares of the adjoining filled figure can be filled with the letters of the word 'ROHINI' so that each row contains at least one letter ? 9360 UD



15. The number of polynomials of the form x^3+ax^2+bx+c that are divisible by x^2+1, when $a, b, c \in \{1, 2, 3, \cdots ; 10\}$ is

(a) 30 (b) 1000 [✓ (c) 10] (d) none of these UD.

16. (i) The number of permutations of the letters of the word AGAIN taken three at a time is

(a) 48 – (b) 24 (c) 36 [✓ (d) 33] UD

(ii) The number of 4 letter words that can be formed using the letters of the word SURITI is

(a) 360 (b) 240 (c) 216 [✓ (d) none of these] UD

(iii) The number of permutations of the letters of the word EXAMINATION taken 4 at a time is

(a) 136 [✓ (b) 2454] (c) 2266 (d) none of these UD

(iv) The number of permutations of the letters of the word EXERCISES taken 5 at a time is

[✓ (a) 2250] (b) 30240 (c) 226960 (d) none of these UD

(v) ~~for~~ The number of 4 letter words that can be formed from the letters of the word PROPORTION is

(a) 700 (b) 750 [✓ (c) 758] (d) none of these UD

17. The number of words that can be formed using letters of the word CALCULATE so that each word starts with and ends with a consonant is

[✓ (a) $\frac{5\underline{|7}}{2}$] (b) $\frac{3\underline{|7}}{2}$ (c) $2\underline{|7}$ (d) none of these UD

18. The number of ways in which lawn-tennis mixed double games can be arranged from 9 married couples if no husband and wife play in the same game is

(a) 756 [✓ (b) 1512] (c) 3024 (d) none of these

19. The number of ways in which three distinct numbers in AP can be selected from 1, 2, 3, ..., 24 is

[✓ (a) 132] (b) 455 (c) $^{12}C_2$ (d) none of these UD

20. Given that n is odd, the number of ways in which three numbers in AP can be selected from 1, 2, 3, ... n is

(a) $\frac{(n-1)^2}{2}$ (b) $\frac{(n+1)^2}{4}$ (c) $\frac{(n+1)^2}{2}$ [✓ (d) $\frac{(n-1)^2}{4}$]

21. ~~Find~~ The sum of all the numbers which can be formed by using the digits 1, 2, 3, 4, 5, when repetition of digits not allowed is

(a) 366000 (b) 660000 (c) 360000 (d) 3999960

22. If 4 digit numbers are formed by using the digits 1, 2, 3, 6, 8, 9, any digit repeated any number of times, then the sum of the numbers formed is

(a) $(1+2+3+6+8+9).\underline{|3}.1111$ [✓ (b) $(1+2+3+6+8+9).6^3.1111$]

(c) $(1+2+3+6+8+9).3^6.1111$ (d) none of these

Greatest value of ^nC_r

$^nC_{\frac{n}{2}}$ is greatest when n is even

$^nC_{\frac{n-1}{2}}$ or $^nC_{\frac{n+1}{2}}$ is greatest when n is odd

23. ~~The~~ Among $^{17}C_5$, $^{17}C_8$, $^{17}C_{10}$, $^{17}C_{11}$, the greatest is

(a) $^{17}C_5$ [✓ (b) $^{17}C_8$] (c) $^{17}C_{10}$ (d) $^{17}C_{11}$

⊗ Some results on combinations

(1) The total number of combinations of n different things taken any number of times at a time

$= {}^nC_0 + {}^nC_1 + \dots + {}^nC_n = 2^n$.

(2) Out of n different things, atleast one (or more) can be chosen in

$2^n - 1$ ways.

(3) If there are $p+q+r$ things, when p things are alike of one kind, q alike and r alike of another kind, then a non-empty selection can be made in

$(p+1)(q+1)(r+1) - 1$ ways.

(4) If there are $p+q+r$ things, p alike one kind, q alike of another kind

and the remaining r things are different, then a non-empty selection can be made in $(p+1)(q+1).2^r-1$ ways.

24. The total number of selections of fruits which can be made from 3 bananas, 4 apples and 2 oranges is

(a) 39 (b) ~~3+~~ 315 (c) 512 [✓ (d) none of these]

25. Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits to each of two persons is

(a) 56 (b) 64 [✓ (c) 66] (d) 72 UD

Division into groups of distinct objects

(1) $m+n$ different things can be divided into two unequal groups containing m and n things $= \frac{\lfloor\underline{m+n}}{\lfloor\underline{m}\,\lfloor\underline{n}}$.

(2) the number of ways in which $2m$ different things can be divided into two equal groups each containing m things $= \frac{\lfloor\underline{2m}}{(\lfloor\underline{m})^2\lfloor\underline{2}}$

Division into groups of identical objects

(1) The number of ways of distribution of n identical objects to r persons (or groups) each one of ~~whose~~ whom can get $0, 1, 2, \ldots, n$ objects $= {}^{n+r-1}C_{r-1}$

(2) the number of ways of distribution on n identical objects to r groups, each one of whom gets atleast one object $= {}^{n-1}C_{r-1}$.

26. The number of ways in which a pack of 52 cards be divided equally among 4 players is

(a) ${}^{52}C_{13}$ (b) ${}^{52}C_4$ [✓ (c) $\frac{\lfloor\underline{52}}{(\lfloor\underline{13})^4}$] (d) $\frac{\lfloor\underline{52}}{(\lfloor\underline{13})^4\lfloor\underline{4}}$

27. The number of ways in which 52 cards can be divided into 4 sets, three of ~~whom~~ them having 17 cards each and fourth one having just one card is

(a) $\frac{\lfloor\underline{52}}{(\lfloor\underline{17})^3}$ [✓ (b) $\frac{\lfloor\underline{52}}{(\lfloor\underline{17})^3\lfloor\underline{3}}$] (c) $\frac{\lfloor\underline{51}}{(\lfloor\underline{17})^3}$ (d) $\frac{\lfloor\underline{51}}{(\lfloor\underline{17})^3\lfloor\underline{3}}$

28. The number of ways in which 9 different balls can be

(i) divided into 5 groups, four of them having 2 balls and fifth group just one ball $\frac{\lfloor\underline{9}}{(\lfloor\underline{2})^4\lfloor\underline{1}\lfloor\underline{4}}$

(ii) ~~distri~~ put into 5 different boxes, four of them containing 2 balls ~~and fifth just one ball~~ and one box just one ball.

29. There are 3 men and 7 women taking a dance class. Find the number of ways in which each man can be paired with a woman partner, and the remaining 4 women can be paired into two pairs.
(Hint. ${}^7C_3 \times \underline{|3} \times \frac{\underline{|4}}{(\underline{|2}\,\underline{|2})\underline{|2}} = 35 \times 6 \times 3 = 630$)

30. Read the passage and answer the following questions.
Five balls are to be placed in 3 boxes. Each box can hold all the five balls. In how many ways can we place the balls so that no box remains empty, when

(i) balls and boxes are different
[✓ (a) 150] (b) 6 (c) 50 (d) 2 UD

(ii) balls are identical but boxes are different
(a) 150 [✓ (b) 6] (c) 50 (d) 2 UD

(iii) balls are different but boxes are identical
(a) 150 (b) 6 [✓ (c) ~~50~~ 25] (d) 2 UD

(iv) balls as well as boxes are identical
(a) 150 (b) 6 (c) 50 [✓ (d) 2] UD.

31. The number of positive integral solution of $abc = 30$ is
(a) 30 [✓ (b) 27] (c) 8 (d) none of these

32. The total number of integral solutions of the equation $xyz = 24$ is
(a) 30 (b) 60 [✓ (c) 120] (d) none of these UD

33. The sum of 5 digit numbers in which only odd digits occur without repetition is
(a) 277775 (b) 555550 (c) 1111100 [✓ (d) none of these] UD

34. The number of ways in which 35 apples can be distributed among 3 students so that each can have any number of apples is
(a) 1332 [✓ (b) 666] (c) 333 (d) none of these

35. The number of ways in which 11 identical pencils can be distributed among 6 kids, each one receiving atleast one is
(a) 168 (b) 308 [✓ (c) 252] (d) none of these

36. If x, y, z are integers and $x \geq 0$, $y \geq 1$, $z \geq 2$ and $x+y+z = 15$, then the number of ordered triplets (x, y, z) is
[✓ (a) 91] (b) 455 (c) ${}^{12}C_2$ (d) none of these UD

37. a, b, c, d are odd natural numbers such that $a+b+c+d=20$, then the number of quadrapulets (a, b, c, d) is

[✓ (a) 165] (b) 455 (c) 310 (d) 255 UD

38. Number of integral solutions of $x+y+z=0$ with $x \geqslant -5$, $y \geqslant -5$, $z \geqslant -5$ is

(a) 134 [✓ (b) 136] (c) 138 (d) 140 UD

39. The number of positive integral solutions of $x+y+z \leq 10$ is

(c) 60 [✓ (b) 120] (c) 240 (d) none of these UD.

40. The number of positive integral solutions of $15 \leq x_1+x_2+x_3 \leq 20$ is

[✓ (a) 685] (b) 785 (c) 1125 (d) none of these UD

41. The number of ordered triplets of positive integers which satisfy the inequality $15 \leq x+y+z \leq 45$ is

UD

(a) ${}^{45}C_2 - {}^{14}C_2$ (b) ${}^{45}C_3 - {}^{15}C_3$ (c) ${}^{46}C_3 - {}^{15}C_3$ (d) none of these

42. Find the number of non-negative integral solutions of $2x+y+z=21$.

132

Solutions

$\frac{2.}{1}$ $E_2(\underline{|100}) = \left[\frac{100}{2}\right] + \left[\frac{100}{4}\right] + \left[\frac{100}{8}\right] + \left[\frac{100}{16}\right] + \left[\frac{100}{32}\right] + \left[\frac{100}{64}\right] + \left[\frac{100}{128}\right] + \cdots$ ①

$= 50 + 25 + 12 + 6 + 3 + 1 + 0 = 97$

$E_5(\underline{|100}) = \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] + \cdots = 20 + 4 + 0 = 24.$

$\therefore \underline{|100} = 2^{97} \times 5^{24} \times \cdots = 2^{24} \times 5^{24} \times 2^{73} \times \cdots$

$= (10)^{24} \times \cdots$

$\frac{3.}{1}$ $E_3(\underline{|100}) = \left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right] + \cdots = 33 + 11 + 3 + 1 + 0$

$= 48$

$\underline{|100} = 2^{97} \times 3^{48} \times \cdots = 2^{96} \times 3^{48} \times \cdots = 4^{48} \times 3^{48} \times \cdots$

$= (12)^{48} \times \cdots$

4\. $E_2(\underline{|24}) = \left[\frac{24}{2}\right] + \left[\frac{24}{4}\right] + \left[\frac{24}{8}\right] + \left[\frac{24}{16}\right] + \cdots = 12 + 6 + 3 + 1 = 22$

$E_3(\underline{|24}) = \left[\frac{24}{3}\right] + \left[\frac{24}{9}\right] + \left[\frac{24}{27}\right] + \cdots = 8 + 2 = 10$

$\underline{|24} = 2^{22} \times 3^{10} \times \cdots = (2^3)^7 \times 3^7 \times \cdots = (24)^7 \times \cdots$

6\. $75600 = 63 \times 12 \times 100 = 2^4 . 3^3 . 5^2 . 7^1$

No of factors $= 5 \times 4 \times 3 \times 2 = 120.$

No of even factors $= 4 \times 4 \times 3 \times 2 = 96$ — proper even factor $= 95$

No of odd factors $= 4 \times 3 \times 2 = 24$ — proper odd factor $= 23$

Sum of divisors $= (1 + 2 + \cdots + 2^4)(1 + 3 + 3^2 + 3^3)(1 + 5 + 5^2)(1 + 7)$

$= 31 \times 40 \times 31 \times 8 = 307520$

Sum of odd divisors $= (1 + 3 + 3^2 + 3^3)(1 + 5 + 5^2)(1 + 7)$

$= 40 \times 31 \times 8 = 9920.$

Sum of proper odd divisors $= 9919$

Sum of proper even divisors $= (2 + 2^2 + \cdots + 2^4)(1 + 3 + 3^2 + 3^3)(1 + 5 + 5^2)(1 + 7) - \underline{75600.}$

$= 30 \times 40 \times 31 \times 8 - 75600$

8\. Sum $= (1 + 2 + 2^2 + \cdots + 2^5)(1 + 3 + \cdots + 3^4)(1 + 5 + 5^2)$

$= 63 \times 121 \times 31 = 3^2 . 7 . 11^2 . 31$

9\. $(6!)^{3!} = (720)^6 = (2^4 . 3^2 . 5)^6 = 2^{24} 3^{12} 5^6$

No. of divisors $= 25 \times 13 \times 7 = 2275.$

10\. $\underline{|5}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} - \frac{1}{\underline{|5}}\right) = \frac{\underline{|5}}{\underline{|2}} - \frac{\underline{|5}}{\underline{|3}} + \frac{\underline{|5}}{\underline{|4}} - \frac{\underline{|5}}{\underline{|5}} = 60 - 20 + 5 - 1 = 44$

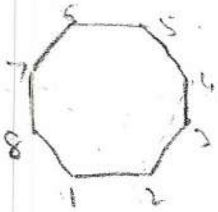
11\. ${}^5C_3\, \underline{|3}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}}\right) = 10\left(\frac{\underline{|3}}{\underline{|2}} - \frac{\underline{|3}}{\underline{|3}}\right) = 10(3 - 1) = 20.$

12\. (i) ${}^{18}C_2 - {}^5C_2 + 1 = 153 - 10 + 1 = 144$

(ii) ${}^{18}C_3 - {}^5C_3 = 816 - 10 = 806$

13\. $\frac{n(n-3)}{2} = 170 \Rightarrow n^2 - 3n - 340 = 0 \Rightarrow (n - 20)(n + 17) = 0$

$\Rightarrow n = 20, \enclose{circle}{-17}$ rejected



14. $^{8}C_3 - 8 - 8 \times {}^{4}C_1 = 56 - 8 - 8\times4 = 16$

(12,23), (23,34), ..., (67,78), (78,81), (81,12)

15. $^{10}C_3$ − (three consecutive) − (two consecutive)

$= {}^{10}C_3 - 10 - 10 \times {}^{6}C_1 = 120 - 10 - 10\times 6 = 50$

abc, bcd, ... hij, ija, jab = 10

16. $1^2+2^2+3^2+\dots+8^2 = \frac{8(8+1)(2\times8+1)}{6} = ~~204~~\ 12\times17 = 204$

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1. $^{7}C_2 \times 2^5 = 21\times32 = 672$

2. No. of numbers between 100 and 1000 = 899

without digit 3 $\quad$ □ □ □ = 648, these include no. 100.
$\qquad\qquad\qquad\quad$ 8 9 9

No. of such numbers between 100 and 1000 = 648 − 1 = 647

Required no. of numbers = 899 − 645 = 252

3. $\overset{3}{□}$ □ □ 4/6 = 12 $\qquad$ $\overset{2/4}{□}$ □ □ 4/6 = 8 $\qquad$ $\overset{4}{□}$ □ □ 4/6 = 8 $\qquad$ 12 + 8 + 8 = 28
$\quad$ 1 4 3 $\qquad\qquad\qquad$ 4 $\qquad\qquad\qquad\qquad$ 1 4 2

⑥ ... 18 $\qquad$ ... ⑥ ... 12 $\qquad$ ... ⑥ ... 12 $\qquad$ 18 + 12 + 12 = 42

24 + 36 = 60.

4. □ □ □ □ $\overset{0}{□}$ = 24 $\qquad$ □ □ □ □ □ 4/4 = 36
$\quad$ 4 3 2 1 1 $\qquad\qquad\qquad$ 3 3 2 1 2

5. Sum of digits = 0+2+4+6+8+9 = 29, so we can exclude either 2 or 8

So 5-digit numbers can be formed by using 0, 4, 6, 8, 9 or 0, ~~4~~ 6, 8, 9

□ □ □ □ □ $\qquad$ Required no. = $(4\times\lfloor 4)\times 2 = 192$
4 4 3 2 1

6. No. is divisible by 4 if the number formed by the last two digits is divisible by 4. $\qquad$ 0, 2, 4, 7, 8, 9

Last two digits may be 04, 08, 24, 28, 20, 48, 40, 72, 84, 80, 92

□ □ □ $\boxed{□\ □}$ 5 (containing zero) = 120 $\qquad$ □ □ □ $\boxed{□\ □}$ 6 numbers (not containing zero)
4 3 2 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 3 3 2 $\qquad$ 18×6 = 108

Total no. of required numbers = 120 + 108 = 228

7. Sum of digits = 0+2+4+5+6+8 = 25

the no. will be divisible by 3 only if 4 excluded.

Thus, we have to form the even numbers using digit 0, 2, 5, 6, 8

□ □ □ □ $\overset{0}{□}$ = 24 $\qquad$ □ □ □ □ □ 2/6/8 $\quad$ 18×3 = 54
4 3 2 1 1 $\qquad\qquad\quad$ 3 3 2 1

Required no. = 78

8. Thousand's place can be filled in 6 ways. Hundred's & ten's place can be filled in 10 ways each. $\qquad$ □ □ □ □
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 6 10 10

No. of nos. formed at these three places = 600. Corresponding to each of these numbers, their sum is either even or odd. There are 5 ways to fill the unit's place to make the sum of all four numbers odd.

600×5 = 3000.

9. $\underset{9}{\square}\ \underset{10}{\square}\ \underset{10}{\square}\ \underset{10}{\square}\ \underset{10}{\square}\ \underset{2}{\square} = 9\times10\times10\times10\times10\times2 = 180000$

10 4,4,4 ; 2,2,2,2 ; 3 3

3 alike – 2 ; 2 alike, 1 differ – ${}^3C_1\times{}^2C_1\times\frac{\lfloor 3}{\lfloor 2} = 3\times2\times3 = 18$

3 differ – $\lfloor 3 = 6$ Total no. $2+18+6 = 26$

11

1, 1, 1, 3, 4, 4, 5

		Odd.
3	$\frac{\lfloor 5}{\lfloor 3\,\lfloor 2} = 10$	5
3	$\frac{\lfloor 5}{\lfloor 2\,\lfloor 2} = 30$	1
4	$\frac{\lfloor 5}{\lfloor 2} = 60$	1
4	$\frac{\lfloor 5}{\lfloor 3} = 20$	3
4	$\frac{\lfloor 5}{\lfloor 3} = 20$	5
5	$\frac{\lfloor 5}{\lfloor 2\,\lfloor 2} = 30$	1
5	$\frac{\lfloor 5}{\lfloor 3\,\lfloor 2} = 10$	3

Total no = 180.

12. – – $\square\ \times\ \times\ \times\ \square$ – – – ${}^{10}C_3\times\lfloor 2\times\lfloor 3\times\lfloor 8$

Total 13 places

13. ${}^{30}C_4 - 27 = \frac{30.29.28.27}{1.2.3.4} - 27 = 27405 - 27 = 27378$

Four consecutive selections can be (1,2,3,4), (2,3,4,5), … , (27, 28, 29, 30)

14. The no. of ways of selecting 6 squares from the figure so that each row contains atleast one letter $= {}^8C_6 - 2 = 28 - 2 = 26$.

Required no. of ways of filling the squares with the letters of the word 'ROHINI' $= 26\times\frac{\lfloor 6}{\lfloor 2} = 26\times360 = 9360$

15. $(x^3+ax^2+bx+c) = (x^2+1)(x+d)$

$\Rightarrow 1=1;\ d=c\ ,\ b=1\ ,\ a=d\ \Rightarrow b=1\ ,\ a=c.$

~~As a,b,c ∈ {1,2,3,…,10} , a = c = 1,2,3,…,10 ; b=1~~

$\underline{x^3+ax^2+x+a}$ as $a\in\{1,2,3,\dots,10\}$

only 10 such polynomials are possible.

16.(i) AA, G, I, N.

Two alike, one other – $1\times{}^3C_1\times\frac{\lfloor 3}{\lfloor 2} = 3\times3 = 9$

3 differ $= {}^4C_3\times\lfloor 3 = 4\times6 = 24$

Total no. of letters permuts = 33

16(ii) S, U, R, T, I I

2 alike, 2 diff. $1 \times {}^4C_2 \times \frac{\lfloor 4}{\lfloor 2} = 6 \times 12 = 72$ Total no = 192

4 different ${}^5C_4 \times \lfloor 4 = 5 \times 24 = 120$

(iii) E, X, AA, M, II, NN, T, O

2 alike, 2 alike (2 pairs) – ${}^3C_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 3 \times 6 = 18$

2 alike, 2 diff. (1 pair) – ${}^3C_1 \times {}^7C_2 \times \frac{\lfloor 4}{\lfloor 2} = 3 \times 21 \times 12 = 756$

4 differen ${}^8C_4 \times \lfloor 4 = 70 \times 24 = 1680$

Total no = $18 + 756 + 1680 = 2454$

(iv) EEE, SS, R, C, I, X

3 alike, 2 alike EEE, SS — $1 \times \frac{\lfloor 5}{\lfloor 3 \lfloor 2} = 10$

3 alike, 2 diff EEE, 2 differen $= 1 \times {}^5C_2 \times \frac{\lfloor 5}{\lfloor 3} = 10 \times 20 = 200$

2 pairs, 1 diff EE, SS, 1 diff $= 1 \times {}^4C_1 \times \frac{\lfloor 5}{\lfloor 2 \lfloor 2} = 4 \times 30 = 120$

2 alike - 3 diff. EE, 3 diff / SS, 3 diff ${}^2C_1 \times {}^5C_2 \times \frac{\lfloor 5}{\lfloor 2} = 2 \times 10 \times 60 = 1200$

5 differen 2 ${}^6C_5 \times \lfloor 5 = 6 \times 120 = 720$

Total no. $= 10 + 200 + 120 + 1200 + 720 = 2250$

(v) PP, RR, OOO, T, N, I

3 alike, 1 diff $= 1 \times {}^5C_1 \times \frac{\lfloor 4}{\lfloor 3} = 5 \times 4 = 20$ Total = 758

2 alike, 2 alike $= {}^3C_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 3 \times 6 = 18$

2 alike, 2 diff $= {}^3C_1 \times {}^5C_2 \times \frac{\lfloor 4}{\lfloor 2} = 3 \times 10 \times 12 = 360$

4 diff $= {}^6C_4 \times \lfloor 4 = 15 \times 24 = 360$

17. CC, LL, T ; AA, U, E

C $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ C

C $\frac{\lfloor 7}{\lfloor 2}$ L

C $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ T

L $\frac{\lfloor 7}{\lfloor 2}$ C

L $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ L

L $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ T

T $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ C

T $\frac{\lfloor 7}{\lfloor 2 \lfloor 2}$ L

Total no = $\frac{\lfloor 7}{\lfloor 2} \left(\frac{1}{2} + 1 + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)$

$= \frac{\lfloor 7}{\lfloor 2} \times 5$ ~~[illegible]~~

18. ${}^9C_2 \times {}^7C_2 \times 2 = 36 \times 21 \times 2 = 1512$

$M_1W_1 \leftrightarrow M_2W_2$; $M_1W_2 \leftrightarrow M_2W_1$

19. Let a, b, c be three numbers in AP, then $a+c=2b$. As $2b$ is even so $a+c$ is also even $\Rightarrow$ the extreme numbers a and c are either both even or both odd. There 12 even nos. and 12 odd numbers.

$${}^{12}C_2 + {}^{12}C_2 = 66+66 = 132$$

20. Let $n = 2m+1$, no. of odd numbers $= m+1$ and no. of even nos $= m$.

$${}^{m+1}C_2 + {}^{m}C_2 = \frac{(m+1)m}{1.2} + \frac{m(m-1)}{1.2} = \frac{m}{2}\{m+1+m-1\} = m^2$$

$$= \left(\frac{n-1}{2}\right)^2 .$$

21. Sum of digits at unit place $= (1+2+3+4+5) \times \lfloor\underline{4}$

(∵ when one particular digit is fixed at unit's place, each of the remaining 4 places can be filled in $\lfloor\underline{4}$ ways)

Similarly, sum of digits in any other place is also the same.

Hence, the sum of all numbers $= (1+2+3+4+5) \times \lfloor\underline{4}\,(1+10+100+1000+10000)$

$= 15 \times 24 \times 11111 =$

22. Sum of digits in the units place $= (1+2+3+6+8+9) \times 6^3$

∵ when one particular digit is fixed at unit's place, then each of the remaining 3 places can be filled in 6 ways).

Similarly sum of digits in all other places is also the same.

∴ Hence, the sum of all numbers $= (1+2+3+6+8+9) \times 6^3 \times (1+10+100+1000)$

$= (1+2+3+6+8+9) \times 6^3 \times 1111$

23. ${}^{17}C_{\frac{17-1}{2}} = {}^{17}C_8$ has greatest

24. Req. no. of selections $= (3+1)(4+1)(2+1) - 1 = 4 \times 5 \times 3 - 1 = 59$

25. Divide 5 apples and 10 oranges in two groups.

No. of ways of selecting 0, 1, 2, …, 15 $= (5+1)(10+1) = 6 \times 11 = 66$.

then distribute oranges ~~&~~ so that each gets 15 fruits.

26. $\dfrac{\lfloor\underline{52}}{(\lfloor\underline{13})\lfloor\underline{4}} \times \lfloor\underline{4} = \dfrac{\lfloor\underline{52}}{(\lfloor\underline{13})^4}$. 13, 13, 13, 13

27. 17, 17, 17, 1 $\dfrac{\lfloor\underline{52}}{(\lfloor\underline{17})^3 \lfloor\underline{1}.\lfloor\underline{3}} = \dfrac{\lfloor\underline{52}}{(\lfloor\underline{17})^3 \lfloor\underline{3}}$

28. (i) $\dfrac{\lfloor\underline{9}}{(\lfloor\underline{2})^4 \lfloor\underline{1}\lfloor\underline{4}} = \dfrac{9.8.7.6.5}{16} = 945$

(ii) $\dfrac{\lfloor\underline{9}}{(\lfloor\underline{2})^4 \lfloor\underline{1}\lfloor\underline{4}} \times \lfloor\underline{5} = 945 \times 120$.

30. the possibilities are 1,1,3 ; 1,2,2

(i) $\frac{\underline{|5}}{\underline{|1}\,\underline{|1}\,\underline{|3}\,\underline{|2}} \times \underline{|3} + \frac{\underline{|5}}{(\underline{|1}\,\underline{|2}\,\underline{|2})\underline{|2}} \times \underline{|3} = 60+90 = 150$

(ii) 1,1,3 ; 1,3,1 ; 3,1,1 ; 1,2,2 ; 2,1,2 ; 2,2,1 = 6 ways

or use $^{n-1}C_{r-1} = {}^{4}C_{2} = \frac{4\times3}{1\times2} = 6$

(iii) $\frac{\underline{|5}}{(\underline{|1}\,\underline{|1}\,\underline{|3})\underline{|2}} + \frac{\underline{|5}}{(\underline{|1}\,\underline{|2}\,\underline{|2})\underline{|2}} = 10+15 = 25$

(iv) 1,1,3 ; 1,2,2 only 2 ways.

31. Different possibilities

30, 1, 1 — $\frac{\underline{|3}}{\underline{|2}} = 3$ Total no. of ways = 27
15, 2, 1 — $\underline{|3} = 6$
10, 3, 1 — $\underline{|3} = 6$
6, 5, 1 — $\underline{|3} = 6$
5, 3, 2 — $\underline{|3} = 6$

is

32 First find positive integral solutions $xyz = 24$

24, 1, 1 → $\frac{\underline{|3}}{\underline{|2}} = 3$ Total = 30.
12, 2, 1 — $\underline{|3} = 6$
8, 3, 1 — $\underline{|3} = 6$
6, 4, 1 → $\underline{|3} = 6$
6, 2, 2 — $\frac{\underline{|3}}{\underline{|2}} = 3$
4, 3, 2 — $\underline{|3} = 6$

For integral solutions any one of x, y, z is +ve, other two −ve.

∴ Required no. of solutions = 30 × 4 = 120.

33. Sum = $(1+3+5+7+9) \times \underline{|4} \times (1+10+100+1000+10000)$
$= 25\times24\times11111 = 600\times11111 = 6666600$

34. Let 3 students get x, y, z apples then $x+y+z = 35$

No. of non-negative integral solutions = $^{35+3-1}C_{3-1} = {}^{37}C_{2}$
$= 37\times18 = 666$

35. Req. no. of ways = $^{11-1}C_{6-1} = {}^{10}C_{5} = \frac{10.9.8.7.6}{1.2.3.4.5} = 252$

36. Let $v = y-1$ and ~~z=~~ $w = z-2$, then $v \geq 0$, $w \geq 0$.

$x+v+w = 12$ No. of solutions = $^{12+3-1}C_{3-1} = {}^{14}C_{2} = \frac{14\times13}{1\times2} = 91$

37. Let $a = 2p+1,\ b = 2q+1,\ c = 2r+1,\ d = 2s+1$ then $p, q, r, s \ge 0$

$(2p+1) + (2q+1) + (2r+1) + (2s+1) = 20 \Rightarrow p+q+r+s = 8.$

No. of solutions $= {}^{8+4-1}C_{4-1} = {}^{11}C_3 = \frac{11.10.9}{1.2.3} = 11 \times 15 = 165$

38. Let $u = x+5,\ v = y+5,\ w = z+5$ then $u, v, w \ge 0$

$(u-5) + (v-5) + (w-5) = 0 \Rightarrow u+v+w = 15.$

No. of solutions $= {}^{15+3-1}C_{3-1} = {}^{17}C_2 = \frac{17 \times 16}{1 \times 2} = 136$

39. Let $x+y+z+a = 10$ where $x \ge 1, y \ge 1, z \ge 1, a \ge 0.$

Let $u = x-1,\ v = y-1,\ w = z-1$ then $u, v, w, a \ge 0$

$u+v+w+a = 7.$

No. of solutions $= {}^{7+4-1}C_{4-1} = {}^{10}C_3 = \frac{10.9.8}{1.2.3} = 120.$

40. $x_1+x_2+x_3+a = 20,\ x \ge 1, y \ge 1, z \ge 1, a \ge 0$

$\Rightarrow u_1+u_2+u_3+a = 17 \qquad u_1, u_2, u_3, a \ge 0$

No of solution $= {}^{17+4-1}C_{4-1} = {}^{20}C_3 = \frac{20.19.18}{1.2.3}$

$= 60 \times 19 = 1140.$

$x_1+x_2+x_3 \le 15,\ x_1 \ge 1, y \ge 1, z \ge 1$

~~$v_1+v_2+v_3 = 12 \quad v_1, v_2, v_3 \ge 0 \quad {}^{12+3-1}C_{3-1} =$~~

$v_1+v_2+v_3+b = 12,\ v_1, v_2, v_3, v_4 \ge 0$

${}^{12+4-1}C_{4-1} = {}^{15}C_3 = \frac{15.14.13}{1.2.3} = 35 \times 13 = 455.$

Req. no of solutions $= 1140 - 455 = \underline{\underline{685}}.$

41. $x+y+z+a = 45,\ x \ge 1, y \ge 1, z \ge 1, a \ge 0$

$u+v+w+a = 42 \qquad u, v, w, a \ge 0$

No of solutions $= {}^{42+4-1}C_{4-1} = {}^{45}C_3$

$x+y+z = 14$

~~$x+y+z+b \le 15 \quad x \ge 1, y \ge 1, z \ge 1 \Rightarrow x+y+z+b = 15$~~

~~$u+v+w = 17,\ u, v, w \ge 0 \quad {}^{17+3-1}C$~~

~~$u+v+w = 11$~~ $\quad 11+$

$x+y+z \le 15,\ x \ge 1, y \ge 1, z \ge 1$

$x+y+z+b = 15,\ x \ge 1, y \ge 1, z \ge 1,\ b \ge 0$

$u+v+w+b = 12, \qquad {}^{12+4-1}C_3 = {}^{15}C_3$

42. Clearly. $x = 0, 1, 2, 3, \ldots, 10.$

$y+z = 21-2h$ where $x = h.$

No of ~~solutions~~ ways of distribution $= {}^{21-2h+2-1}C_{2-1} = {}^{22-2h}C_1$

$= 22-2h.$

Req. No of soln $= \sum_{h=0}^{10} (22-2h) = 22+20+18+\cdots+2$

$= 2(1+2+\cdots+11) = 2.\frac{11 \times 12}{2} = 132.$